DOI:
10.1109/TFUZZ.2019.2955044

Fuzzy-Model-Based Output Feedback Steering Control in Autonomous Driving Subject to Actuator Constraints

Changzhu Zhang, *Member, IEEE*, Hak-Keung Lam, *Senior Member, IEEE*, Jianbin Qiu, *Senior Member, IEEE*, Peng Qi, and Qijun Chen

***Abstract*—In this paper, the problem of steering control based on T-S fuzzy vehicle lateral dynamics is investigated for autonomous driving with nonlinearities, system uncertainties, and actuator constraints. During normal vehicle cruising, the vehicle velocity always changes due to the different road conditions and/or steering wheel maneuvers, and moreover, the vehicle dynamics is also significantly influenced by the tire/road forces under different road surface conditions, which brings many difficulties in steering controller design. By adopting fuzzy modeling techniques and varying look-ahead control strategy, an approach to the T-S fuzzy anti-windup output feedback controller design is proposed for the steering control in path tracking within T-S fuzzy-model-based analysis framework, where the actuator amplitude saturation and rate limit are simultaneously taken into consideration. Finally, valuation results with Carsim/Matlab joint simulation are shown to demonstrate the effectiveness of the developed methods, and some comparison results in path tracking performance with fixed look-ahead distance control rule and the driver model controller embedded in Carsim are provided, which illustrate the advantages of the developed controller design method.**

***Index Terms*—Autonomous driving, actuator constraints, output feedback, fuzzy control, Carsim/Matlab joint simulation.**

I. INTRODUCTION

The past few years have witnessed the rapid development of the autonomous driving technologies both in academical and industrial communities. On the one hand with full autonomy, autonomous driving is expected to increase comfort, optimize fuel consumption, reduce pollution emission, and most importantly, enhance traffic safety, etc. [1], [2]. On other hand, when traveling with autonomous vehicles, it is not necessary for the drivers to focus their attention on driving, which is usually boring and exhausting, especially in long distance journeys, and instead the drivers have the possibility to do something else with time otherwise spent on driving, which might greatly improve societal and economic benefits. An autonomous vehicle refers to self-acting and self-regulating car that can guide itself, familiarize itself with surroundings, make decisions, and fully handle all possible situations without any human intervention. Generally speaking, the structure of autonomous driving technology consists of environment perception and localization, decision making and path planning, and vehicle control.

In past few decades, much research attention has been paid to perception, localization, planning, and decision making since the similar topics have been widely investigated in robotics [3]–[5]. Recently, as one of the most important problems in autonomous driving technology, the steering control or path tracking control has attracted much research attention in control area, which is focused on the control algorithm designs such that the self-driving cars can follow a reference trajectory. To be more specific, the steering or path tracking control is to keep the self-driving cars running along the reference trajectories with respect to different vehicle velocity, road curvature and surface conditions, and vehicle nonlinearities and parametric uncertainties, etc. In the existing literature, there have been some remarkable results reported on the problem of steering or path tracking control, such as, PID control [6]–[9], fuzzy logic or fuzzy model-based control [10]–[15], model predictive control [16]–[19], and robust nonlinear control [20]–[26].

In practice, some physical states of the vehicle are unavailable, such as lateral speed and acceleration, and the accurate vehicle dynamics cannot be obtained, and thus it is very difficult for the steering controller synthesis with vehicle model-based control methods. Therefore, a few results have been proposed with the classical PID control strategies at the early stage of steering control study. To mention a few, a self-tuning PID controller for steering control in urban traffic environment was studied in [7], and two PID parameter tuning mechanisms were considered, that is, one rule to minimize the quadratic performance index of the parameter to be tuned and an alternative one was used to reduce the risk of system instability compared with the former one. A cascade PID control scheme for vision-based path following was studied and experimentally tested in [9], and this type of control strategy

This work was supported in part by the National Natural Science Foundation of China (U1764261, 61733013, 61922063, 61873311, 61872271, 61763045), in part by the Fundamental Research Funds for the Central Universities, in part by the Self-Planned Task of State Key Laboratory of Robotics and Systems of Harbin Institute of Technology (SKLRS201801A03), in part by the Natural Science Foundation of Shanghai (19ZR1478900), and in part by China Scholarship Council. (*Corresponding Author*: Peng Qi)

C. Zhang, P. Qi, and Q. Chen are with the Department of Control Science and Engineering, Tongji University, Shanghai 201804, China, and C. Zhang also with the Department of Informatics, King's College London, Bush House, Strand campus, 30 Aldwych, London, WC2B 4BG, United Kingdom. (Email: zhuziy1ster@gmail.com, pqi@tongji.edu.cn, qjchen@tongji.edu.cn)

H. K. Lam is with the Department of Informatics, King's College London, Bush House, Strand campus, 30 Aldwych, London, WC2B 4BG, United Kingdom. (Email: hak-keung.lam@kcl.ac.uk)

J. Qiu is with the State Key Laboratory of Robotics and Systems, Harbin Institute of Technology, and also with the Research Institute of Intelligent Control and Systems, Harbin Institute of Technology, Harbin 150080, China. (Email: jbqiu@hit.edu.cn)

leads to a nested independent control loops architecture which facilitates the PID controller design for multivariable physical plants. There are also some early results on steering controller designs with classical control theory in the existing literature. In [27], a controller design for lateral path tracking was studied by the root locus method with a path-dependent coordinate system and several specified requirements are verified for various designs with different compensation schemes. With the pole placement approach, the problem of yaw rate control for car steering was investigated in [24]. To deal with the nonlinearities and parametric uncertainties, some approaches to the steering control within fuzzy control framework are developed. In [11], a fuzzy logic control for path tracking with a fuzzy preview rule has been developed by considering the upcoming road curvature. However, it should be noted that if forward and backward driving under low speed are required on a complex reference trajectory, the precise control becomes much more difficult. Thus, Wang et al. in [13] presented a mathematical model of the reference path with a sensing system, which shows the effectiveness of the tracking control with different reference paths. Recently, Nguyen et al. studied the problem of automatic lane keeping control by considering multiple system constraints with T-S fuzzy model-based method, and some simulation and experimental results were provided to validate the effectiveness of the proposed methods [15]. To deal with the system constraints in steering control, such as, the control inputs and system states, model predictive control strategies were studied for steering control problem in [17] and [18] based on nonlinear lateral vehicle dynamics, where the nonlinear optimization problems are required to be solved online, and thus the real-time implementation is difficult to be guaranteed, especially for high speed vehicles. To tackle the variations in velocity, mass, and road-tire contact, robust linear and nonlinear design methods were developed in [25] with the measured information of lateral offset and yaw rate.

From the literature review, it can be seen that these aforementioned works have significant importance on both theoretical advancement and practical applications on steering control for autonomous vehicles. However, it still leaves much to be desired. It should be noted that, in most of the existing controller design results with vehicle dynamics, it is commonly assumed that some of the physical vehicle parameters are constant, such as the vehicle velocity, during the normal cruising. However, it is not the case in practice as these parameters usually vary with the different steering wheel input or road conditions accordingly. Moreover in vehicle motions, the tire/road forces are of great significance and the knowledge of the tire/road force dynamics is crucial. In fact, it is demonstrated that a strong nonlinear relationship exists in the force dynamics under different road surface conditions and driving manoeuvres, which is difficult or even impossible to be characterized mathematically. Fortunately as an effective way to tackle these nonlinear dynamics, fuzzy systems have been well studied in the past few years [28]–[31]. In particular, it has been shown that with the approaches in [28], [29], the path to the equilibrium can be arbitrarily imposed, and with the methods in [30], [31], the robustness property can be greatly improved. Moreover, there are several semi-empirical descriptions for road/tire forces proposed in automobile engineering area, such as, Burckhardt model [32], Magic formula [33], and Dugoff model [34], however, all the aforementioned models possess severe nonlinearities and thus cause additional difficulties in system analysis. Furthermore, in practical autonomous driving platform, the electric motors are usually used as the actuators to drive the steering wheel, which are subject to some constraints, such as, amplitude saturation and rate limit. However, most existing works did not take these constraints into account in the controller design procedure, which may lead to serious deterioration of the control performance or even system instability. Based on the aforementioned discussions, it is noted that, though there have been many research works on steering control reported in the literature, the problem of controller design with vehicle lateral dynamic subject to parametric uncertainties, nonlinearities, and actuator constraints has not been fully addressed yet. All these are motivations for our present work.

In current work, we consider the problem of robust steering control for path tracking by taking system nonlinearities, uncertainties, and actuator constraints into account. In particular, a T-S fuzzy representation for the lateral look-ahead tracking dynamics is first developed by considering the parameter variations and nonlinearities. Based on the T-S fuzzy modeling approach, we will develop an anti-windup output feedback controller via parallel distributed compensation (PDC) scheme for the path tracking system with actuator amplitude and rate saturations. Finally, a design method to the corresponding controller is proposed by solving a convex optimization problem subject to several linear matrix inequality conditions, which can be implemented in Matlab with LMI Toolbox. The main contributions of this paper are summarized as follows. 1) With the look-ahead strategy, a T-S fuzzy vehicle dynamics for steering control is developed by considering the parameter variations and a time-varying preview distance rule for the controller design is first proposed, which mainly depends on the vehicle velocity in practical driving. 2) A T-S fuzzy anti-windup dynamic output feedback compensator for the nonlinear lateral tracking system is derived to deal with the actuator constraints, i.e., amplitude saturation and rate limit. 3) To verify the effectiveness of the proposed controller design, a prototype D-Sedan in Carsim is utilized as the autonomous vehicle to follow a reference trajectory and the controller is realized in Simulink/Matlab. With some experimental results in such Carsim/Matlab joint simulation environment, it clearly shows the effectiveness and advantages of the designed controller over the fixed preview distance control strategy and the driver model in Carsim in terms of tracking performance.

The structure of the work is given as follows. In Section II, a T-S fuzzy vehicle model for steering control and the problem to be solved are formulated. In Section III, the dynamic output feedback compensator is developed and the main results are presented. The experimental setup and some simulation results are given in Section IV. We finally summarize the paper in Section V.

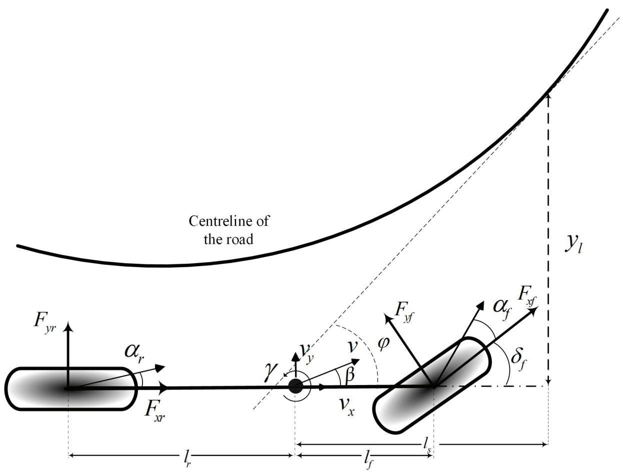


Fig. 1. Look-ahead path tracking based on single-track model

II. System Modeling and Problem Formulation

In general, the behaviors of vehicle dynamics exhibit strong nonlinearities, which lead to great challenges to the relevant vehicle model-based controller design. To facilitate the steering system analysis and output feedback controller synthesis, the widely used single-track vehicle dynamic model for path tracking is shown in Figure 1.

A. Vehicle Lateral Dynamics

Before presenting the model of the lateral vehicle dynamics, we introduce the nomenclature list as follows:

NOMENCLATURE

m	vehicle mass (kg).
f, r	subscripts denoting front and rear.
J_z	vehicle yaw moment of inertia (kg·m^2).
$l_{f(r)}$	distance of CG from front (rear) axle.
F_{xi}	longitudinal tire/road forces (N) ($i = f, r$).
F_{yi}	lateral tire/road forces (N) ($i = f, r$).
$C_{\alpha i}$	cornering stiffness (N/rad) ($i = f, r$).
$\alpha_{f,r}$	front/rear tire slip angle (rad).
v_x	vehicle longitudinal vehicle (m/s).
v_y	vehicle lateral vehicle (m/s).
β	vehicle side slip angle (rad).
γ	vehicle yaw rate (rad/s).
δ_f	front road wheel steering angle (rad).
φ	relative yaw angle (rad).
l_s	preview distance (m).
d	radius of road curves (m).
w	road curvature.
$\mathbb{R}^{m\times n}$	m by n real matrix.
$\mathbb{S}^n$	n by n real symmetric matrix.

In present work, we consider the dynamics of the front-wheel driven vehicles, and thus the rear longitudinal force F_{xr} can be neglected. In this case, the vehicle motion behavior is characterized by the following mathematical equations:

$$\begin{aligned} m(\dot{v}_x - \gamma v_y) &= F_{xf}\cos\delta_f - F_{yf}\sin\delta_f, \\ m(\dot{v}_y + \gamma v_x) &= F_{xf}\sin\delta_f + F_{yf}\cos\delta_f + F_{yr}, \\ J_z\dot{\gamma} &= l_f(F_{xf}\sin\delta_f + F_{yf}\cos\delta_f) - l_r F_{yr}, \end{aligned} \quad (1)$$

which can be obtained from the structure of single-track model shown in Figure 1.

Since we are concerning with the steering control along the centerline of the road which can be characterized by the last two equations in (1), it is commonly assumed that the longitudinal vehicle velocity control is achieved by a pre-designed velocity controller, and thus the longitudinal dynamics given by the first equation is out of our research scope. By considering the kinematics relationship $v_y = v_x \tan\beta$ and small angle approximation, the vehicle lateral dynamics can be rewritten as:

$$\begin{aligned} mv_x(\dot{\beta} + \gamma) &= F_{yf} + F_{yr}, \\ J_z\dot{\gamma} &= l_f F_{yf} - l_r F_{yr}. \end{aligned} \quad (2)$$

With small tire slip angles, the lateral tire/road forces F_{yf} and F_{yr} can be linearly approximated by the following functions [43]:

$$F_{yf} = -2C_{\alpha f}\alpha_f, F_{yr} = -2C_{\alpha r}\alpha_r, \quad (3)$$

where α_f and α_r are given by $\beta + \frac{\gamma l_f}{v_x} - \delta_f$ and $\beta - \frac{\gamma l_r}{v_x}$, respectively.

In most of the existing results, to facilitate the following controller design, linear lateral tire/road force representation is widely adopted in lateral force dynamics modeling. However, the lateral forces cannot be simply described by the constant cornering stiffness and the slip angles as the lateral forces also depend on other variables, such as, tire temperature, tire pressure, and road characteristics [35]. Therefore, to more accurately represent the lateral dynamics of path tracking, uncertain cornering stiffness varying in an appropriate interval is utilized to compensate the nonlinearity of lateral forces [36],

$$\begin{aligned} F_{yf} &= -2(C_{\alpha f} + \triangle C_{\alpha f}(\bullet))\alpha_f, \\ F_{yr} &= -2(C_{\alpha r} + \triangle C_{\alpha r}(\bullet))\alpha_r, \end{aligned} \quad (4)$$

where $C_{\alpha i} = \frac{\max(C_{\alpha i}(\bullet)) + \min(C_{\alpha i}(\bullet))}{2}$, $\triangle C_{\alpha i}(\bullet) \in [-\triangle C_{\alpha i}, \triangle C_{\alpha i}]$, $\triangle C_{\alpha i} = \frac{\max(C_{\alpha i}(\bullet)) - \min(C_{\alpha i}(\bullet))}{2}$ $(i = f, r)$, $C_{\alpha i}(\bullet)$ denotes the uncertain cornering stiffness, and $(\bullet)$ represents all possible variables causing its variations.

For the path tracking problem in autonomous driving, it is of great significance to obtain the relative distance between the vehicle position and the road to follow, i.e., the path tracking error or lateral offset. In general, several methods are utilized to compute the tracking error, such as, GPS, SLAM via Lida sensor and/or camera [38]. As the present work is focused on steering control for path tracking in lower-level layer in the structure of autonomous driving, we assume that the tracking errors in a preview distance l_s are available as computed by upper perception module, and its dynamics can be expressed as [21], [25]:

$$\dot{y}_l(t) = \beta v_x + l_s\gamma + v_x\varphi. \quad (5)$$

During the normal vehicle cruising, it is ineluctable that the vehicle velocity varies according to different road conditions. In this paper, we assume $v_{x\min} \leq v_x(t) \leq v_{x\max}$, where $v_{x\min}$

and $v_{x\max}$ are constants. Now, the lateral dynamics of path tracking with time-varying parameters is readily obtained by combining equations (2)-(5):

$$\begin{aligned}\dot{x}(t) = (\vec{A}(t) + \triangle\vec{A}(t))x(t) + (\vec{B}(t) + \triangle\vec{B}(t))\delta_f(t) \\ + \vec{D}(t)w(t)\end{aligned} \tag{6}$$

where $x(t) = \left[\beta(t)\ \gamma(t)\ \varphi(t)\ y_l(t)\right]^T \in \mathcal{R}^{n_x}$, $w(t) = 1/d(t)$, and

$$\vec{A}(t) = \begin{bmatrix} \frac{-2C_{\alpha f}-2C_{\alpha r}}{mv_x} & \frac{2l_rC_{\alpha r}-2l_fC_{\alpha f}}{mv_x^2}-1 & 0 & 0 \\ \frac{2l_rC_{\alpha r}-2l_fC_{\alpha f}}{J_z} & \frac{-2l_f^2C_{\alpha f}-2l_r^2C_{\alpha r}}{J_zv_x} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ v_x & l_s & v_x & 0 \end{bmatrix},$$

$$\vec{B}(t) = \begin{bmatrix} \frac{2C_{\alpha f}}{mv_x} \\ \frac{2l_fC_{\alpha f}}{J_z} \\ 0 \\ 0 \end{bmatrix}, \vec{D}(t) = \begin{bmatrix} 0 \\ 0 \\ -v_x \\ 0 \end{bmatrix}. \tag{7}$$

With the expressions of lateral tire/road forces in (4), the uncertain terms $\triangle\vec{A}(t)$ and $\triangle\vec{B}(t)$ can be derived as $\triangle\vec{A}(t) = \vec{R}(t)F(t)\vec{S}_1(t)$ and $\triangle\vec{B}(t) = \vec{R}(t)F(t)\vec{S}_2(t)$, respectively, where

$$\vec{R}(t) = \begin{bmatrix} \frac{2\Delta C_{\alpha f}}{mv_x} & \frac{2\Delta C_{\alpha r}}{mv_x} \\ \frac{2l_f\Delta C_{\alpha f}}{J_z} & \frac{-2l_r\Delta C_{\alpha r}}{J_z} \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\vec{S}_1(t) = \begin{bmatrix} -1 & -\frac{l_f}{v_x} & 0 & 0 \\ -1 & \frac{l_r}{v_x} & 0 & 0 \end{bmatrix}, \vec{S}_2(t) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \tag{8}$$

and $F(t) : \mathbf{Z}^+ \to \mathbb{R}^{n_1 \times n_2}$ represents an unknown time-varying matrix function satisfying $F^T(t)F(t) \le I_{n_2}$.

Remark 2.1: From the modeling procedure it can be observed that the nonlinear lateral vehicle dynamics is simplified via small angle approximations and linear relationships with uncertain cornering stiffness. On the other hand, a look-ahead control rule is adopted with a preview distance l_s. However, the vehicle velocity varies during the cruising, and thus a varying preview distance is considered in this work, that is, a longer preview distance is selected as the velocity increases, which seems more reasonable according to the practical driving experience. To this end, we assume that the preview distance is determined by T_pv_x, where T_p is a prescribed preview time.

For the output feedback controller design in path tracking, we have to define the system output and regulated output equations. With the sensors installed in the autonomous driving platform, some physical information of the vehicle dynamics can be directly measured, such as, the yaw rate and the heading angle. Furthermore, the relative yaw angle φ and the lateral distance offset l_s can be obtained with respect to the reference trajectory. Thus we have the following measured output and regulated output equations,

$$y(t) = Cx(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x(t), \tag{9}$$

$$z(t) = Hx(t) = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} x(t), \tag{10}$$

where $y(t) \in \mathcal{R}^{n_y}$ and $z(t) \in \mathcal{R}^{n_z}$.

B. Fuzzy Modeling for Vehicle Lateral Dynamics

To ease the implementation of the steering control algorithm in the platform of autonomous driving based on Electronic Control Unit, we discretize the continuous-time dynamics with sampling period T_s. Obviously, from (6) we can observe that some matrix parameters depend on the vehicle velocity $v_x(t)$. In present work, we resort to the fuzzy modeling method to deal with the time-varying lateral vehicle dynamics. With the assumption $v_{x\min} \le v_x(t) \le v_{x\max}$ and by resorting to the classical sector nonlinearity method, a discrete-time fuzzy representation of lateral dynamics can be obtained as follows [37]:

Fuzzy Rule $\mathbf{R}^i$: If $\varsigma_1(k)$ is $\mathcal{F}_1^i$ and $\varsigma_2(k)$ is $\mathcal{F}_2^i$ and $\varsigma_3(k)$ is $\mathcal{F}_3^i$, **Then**

$$\begin{cases} x(k+1) = (A_i + \triangle A_i)x(k) + (B_i + \triangle B_i)\delta_f(k) + D_iw(k) \\ y(k) = Cx(k) \\ z(k) = Hx(k), i = 1, 2, \ldots, r, \end{cases} \tag{11}$$

where $\varsigma_1(k) = v_x(k)$, $\varsigma_2(k) = \frac{1}{v_x(k)}$, and $\varsigma_3(k) = \frac{1}{v_x^2(k)}$ are premise variables, $\mathcal{F}_j^i$ is the fuzzy set, and the system matrices $A_i, B_i, D_i, \triangle A_i$, and $\triangle B_i$ are the discretized forms of those defined in (7) and (8) with parameters $v_x(k)$ being replaced by $v_{x\min}$ and $v_{x\max}$, respectively.

Let $\mu_i(\varsigma(k))$ be the normalized membership function of $\mathcal{F}^i$, where $\varsigma(k) = [\varsigma_1(k), \varsigma_2(k), \varsigma_3(k)]$ denotes the premise variable vector, and $\mathcal{F}^i$ is given by $\mathcal{F}^i = \prod_{j=1}^3 \mathcal{F}_j^i$ and $\sum_{i=1}^r \mu_i(\varsigma(k)) = 1$. For brevity, we denote $\mu_i(\varsigma(k))$ by μ_i. With the condition $v_{x\min} \le v_x(t) \le v_{x\max}$, we have

$$\begin{aligned} \mathcal{F}_{1\max} &= \frac{(\varsigma_{1\max} - \varsigma_1)}{(\varsigma_{1\max} - \varsigma_{1\min})}, \mathcal{F}_{1\min} = \frac{(\varsigma_1 - \varsigma_{1\min})}{(\varsigma_{1\max} - \varsigma_{1\min})} \\ \mathcal{F}_{2\max} &= \frac{(\varsigma_{2\max} - \varsigma_2)}{(\varsigma_{2\max} - \varsigma_{2\min})}, \mathcal{F}_{2\min} = \frac{(\varsigma_2 - \varsigma_{2\min})}{(\varsigma_{2\max} - \varsigma_{2\min})} \\ \mathcal{F}_{3\max} &= \frac{(\varsigma_{3\max} - \varsigma_3)}{(\varsigma_{3\max} - \varsigma_{3\min})}, \mathcal{F}_{3\min} = \frac{(\varsigma_3 - \varsigma_{3\min})}{(\varsigma_{3\max} - \varsigma_{3\min})} \end{aligned}$$

where $\varsigma_{1\max}$, $\varsigma_{2\max}$, $\varsigma_{3\max}$, $\varsigma_{1\min}$, $\varsigma_{2\min}$, and $\varsigma_{3\min}$ are the maximal and minimal values of $\varsigma_1(k)$, $\varsigma_2(k)$, and $\varsigma_3(k)$, and $\mathcal{F}_j^i (i = 1\ldots, 8, j = 1, 2, 3)$ can be determined by these functions given above. Thus, $\mathcal{F}^i$ can be obtained by the 2^3 combinations of $\mathcal{F}_1^i$, $\mathcal{F}_2^i$, and $\mathcal{F}_3^i$. Using a standard fuzzy-inference approach, i.e., a singleton fuzzifier, product fuzzy inference, and center-average defuzzifier, yields the following compact presentation of (11)

$$\begin{cases} x(k+1) = \bar{A}(\mu)x(k) + \bar{B}(\mu)\delta_f(k) + D(\mu)w(k) \\ y(k) = Cx(k) \\ z(k) = Hx(k) \end{cases} \tag{12}$$

where

$$\begin{cases} \bar{A}(\mu) = A(\mu) + \Delta A(\mu) = \sum_{i=1}^r \mu_i(A_i + \Delta A_i) \\ \bar{B}(\mu) = B(\mu) + \Delta B(\mu) = \sum_{i=1}^r \mu_i(B_i + \Delta B_i) \\ D(\mu) = \sum_{i=1}^r \mu_iD_i, \Delta A_i = R_iF(k)S_{1i} \\ \Delta B_i = R_iF(k)S_2, \mu := \mu(k) = [\mu_1, \cdots, \mu_r]. \end{cases}$$

From the fuzzy modeling procedure, it is noted that a T-S fuzzy representation of lateral path tracking dynamics is derived via sector nonlinearity modeling method with premise variables $v_x(k)$, $\frac{1}{v_x(k)}$, and $\frac{1}{v_x^2(k)}$. For bounded $v_x(k) \in$

$[v_{x\min}, v_{x\max}]$, $r = 2^3$ linear subsystems are used to smoothly approximate the original discrete-time system with membership functions. Inspired by [21] by exploiting the relationship among these premise variables, the numerical computation of controller design and its real-time implementation can be further improved since the number of linear subsystems is significantly reduced. Defining a scalar variable $\varsigma_v \in [-1, 1]$ that describes the change of $v_x(k)$ between the bounds $v_{x\min}$ and $v_{x\max}$, one has

$$\begin{aligned} \frac{1}{v_x} &= \frac{1}{v_0} + \frac{1}{v_1}\varsigma_v, v_x \cong v_0(1 - \frac{v_0}{v_1}\varsigma_v), \\ \frac{1}{v_x^2} &\cong \frac{1}{v_0^2}(1 + 2\frac{v_0}{v_1}\varsigma_v), \end{aligned} \tag{13}$$

and

$$v_0 = \frac{2v_{x\,\min}v_{x\,\max}}{v_{x\,\min} + v_{x\,\max}}, v_1 = \frac{2v_{x\,\min}v_{x\,\max}}{v_{x\,\min} - v_{x\,\max}}.$$

Furthermore, by setting $\varsigma_v = -1$, we have $v_x(k) = v_{x\min}$, and $\varsigma_v = 1, v_x(k) = v_{x\max}$. In this way, it can be easily found that the number of subsystems is greatly reduced from 8 to 2, and thus, the computation burden of controller parameters and its real-time implementation could be relaxed as expected.

C. Anti-windup Output Feedback Controller

For the automatic steering system in autonomous driving, the motors are usually utilized as actuator to drive the steering wheel, which are inevitably subject to amplitude and rate limitations due to mechanical restrictions in practice. Now, we assume that the steering angle input $\delta_f(k)$ is subject to the following constraints:

- Amplitude saturation: $|\delta_f(k)| \le \rho_a$, and constant scalar $\rho_a > 0$ denotes the control amplitude bound;
- Rate limit: $\Delta\delta_f(k) = |\delta_f(k) - \delta_f(k-1)| \le \rho_r$, and $\rho_r > 0$ denotes the control rate bound.

Based on the aforementioned discussion, the output measurement $y(k)$ is available for controller. Hence, we are in the position to present the T-S fuzzy output feedback compensator with input saturating integrators as follows [39]:

Fuzzy Rule $\mathbf{R}^i$: **If** $\varsigma_1(k)$ is $\mathcal{F}_1^i$ and $\varsigma_2(k)$ is $\mathcal{F}_2^i$ and $\varsigma_3(k)$ is $\mathcal{F}_3^i$, **Then**

$$\begin{cases} v(k+1) = I_{n_u}v(k) + \mathrm{sat}_r(y_c(k)) \\ x_c(k+1) = A_i^c x_c(k) + B_i^c \begin{bmatrix} y(k) \\ v(k) \end{bmatrix} \\ \qquad\qquad + E_i^{ca}(\mathrm{sat}_a(v(k)) - v(k)) \\ \qquad\qquad + E_i^{cr}(\mathrm{sat}_r(y_c(k)) - y_c(k)) \\ y_c(k) = C_i^c x_c(k) + D_i^c \begin{bmatrix} y(k) \\ v(k) \end{bmatrix}, i = 1, 2, \ldots, r, \end{cases} \tag{14}$$

where $x_c(k) \in \mathcal{R}^{n_x+n_u}$ and $y_c(k) \in \mathcal{R}^{n_u}(n_u = 1)$ are, respectively, the state and the output vectors of the controller, and $v(k) \in \mathcal{R}^{n_u}$ denotes the system state of the integrator and the control input of (11), $\mathrm{sat}_a(\cdot)$ and $\mathrm{sat}_r(\cdot)$ represent the saturation functions on control amplitude and rate. Matrices A_i^c, B_i^c, C_i^c, and D_i^c are controller gains with appropriate dimensions, and E_i^{ca} and E_i^{cr} are anti-windup gains ($i \in \{1, 2, \ldots, r\}$). Similarly, the compact form is given as follows:

$$\begin{cases} x_c(k+1) = A^c(\mu)x_c(k) + B^c(\mu)\begin{bmatrix} y(k) \\ v(k) \end{bmatrix} \\ \qquad\qquad + E^{ca}(\mu)(\mathrm{sat}_a(v(k)) - v(k)) \\ \qquad\qquad + E^{cr}(\mu)(\mathrm{sat}_r(y_c(k)) - y_c(k)) \\ y_c(k) = C^c(\mu)x_c(k) + D^c(\mu)\begin{bmatrix} y(k) \\ v(k) \end{bmatrix} \end{cases} \tag{15}$$

and the matrices involved are

$$\begin{aligned} A^c(\mu) &= \sum_{i=1}^r \mu_i A_i^c, B^c(\mu) = \sum_{i=1}^r \mu_i B_i^c, \\ E^{ca}(\mu) &= \sum_{i=1}^r \mu_i E_i^{ca}, C^c(\mu) = \sum_{i=1}^r \mu_i C_i^c, \\ E^{cr}(\mu) &= \sum_{i=1}^r \mu_i E_i^{cr}, D^c(\mu) = \sum_{i=1}^r \mu_i D_i^c. \end{aligned}$$

With the controller given above, the actual control inputs are constrained in terms of amplitude and rate limitations, that is, $\delta_f(k) = \mathrm{sat}_a(v(k))$ and $\Delta\delta_f(k) = \mathrm{sat}_r(y_c(k))$, where

$$\mathrm{sat}_a(v(k)) = \begin{cases} \rho_a, & \text{if } v(k) > \rho_a \\ v(k), & \text{if } -\rho_a \le v(k) \le \rho_a \\ -\rho_a, & \text{if } v(k) < -\rho_a \end{cases} \tag{16}$$

$$\mathrm{sat}_r(y_c(k)) = \begin{cases} \rho_r, & \text{if } y_c(k) > \rho_r \\ y_c(k), & \text{if } -\rho_r \le y_c(k) \le \rho_r \\ -\rho_r, & \text{if } y_c(k) < -\rho_r. \end{cases} \tag{17}$$

According to the presentation of dynamic output feedback controller given above and the property of saturation function, it yields that

$$\begin{aligned} |\Delta\delta_f(k)| &= |\mathrm{sat}_a(v(k) + \mathrm{sat}_r(y_c(k))) - \mathrm{sat}_a(v(k))| \\ &\le |\mathrm{sat}_r(y_c(k))| \le \rho_r. \end{aligned} \tag{18}$$

Hence, with the structure of output feedback controller in (14), the control signal $\delta_f(k)$ also follows the rate constraint inherently.

D. Robust $\mathcal{H}_\infty$ Control Problem Formulation

For a given scalar $\lambda > 0$, design a dynamic output feedback compensator in (14) satisfying: 1) the path tracking system in (6) with uncertainties and actuator constraints can asymptotically follow the given path with zero offset under the output feedback controller (14) when the road curvature is 0, i.e., $w(k) = 0$; 2) under zero initial condition, the following inequality holds

$$\|z(k)\|_{l_2} \le \lambda \|w(k)\|_{l_2} \tag{19}$$

for any different types of path with $w(k)$ belonging to $l_2[0, \infty)$.

For subsequent use, we introduce the following lemma.

Lemma 2.1 [41]: For matrices $\mathcal{X}$, $\mathcal{L}$, and $\mathcal{K}$ of appropriate dimensions and $\mathcal{X} = \mathcal{X}^T$, the following inequality:

$$\mathcal{X} + \mathcal{L}F(k)\mathcal{K} + \mathcal{K}^T F^T(k)\mathcal{L}^T < 0$$

holds for all $F(k)$ satisfying $F^T(k)F(k) \le I$ if and only if there exists $\varepsilon > 0$ such that

$$\Gamma_1 + \varepsilon^{-1}\mathcal{L}\mathcal{L}^T + \varepsilon\mathcal{K}^T\mathcal{K} < 0.$$

III. MAIN RESULTS

In this section, we consider the problem of output feedback control for path tracking with uncertainties and actuator constraints. To this end, we first reformulate the lateral vehicle dynamics and output feedback controller to derive the closed-loop path tracking system. Define augmented vectors $\tilde{x}(k) = \begin{bmatrix} x^T(k) & v^T(k) \end{bmatrix}^T$ and $\tilde{y}(k) = \begin{bmatrix} y^T(k) & v^T(k) \end{bmatrix}^T$, and then one has

$$\begin{cases} \tilde{x}(k+1) = \bar{\mathbf{A}}(\mu)\tilde{x}(k) - \bar{\mathbf{B}}(\mu)\phi_a(K_a\tilde{x}(k)) \\ \qquad\qquad + \mathbf{L}\mathrm{sat}_r(y_c(k)) + \mathbf{D}(\mu)w(k) \\ \tilde{y}(k) = \mathbf{C}\tilde{x}(k) \end{cases} \tag{20}$$

where

$$\begin{cases} \bar{\mathbf{A}}(\mu) = \mathbf{A}(\mu) + \Delta\mathbf{A}(\mu), \bar{\mathbf{B}}(\mu) = \mathbf{B}(\mu) + \Delta\mathbf{B}(\mu) \\ \Delta\mathbf{A}(\mu) = \begin{bmatrix} \Delta A(\mu) & \Delta B(\mu) \\ 0_{n_u\times n_x} & I_{n_u} \end{bmatrix} \\ \mathbf{D}(\mu) = \begin{bmatrix} D(\mu) \\ 0_{n_u\times n_w} \end{bmatrix}, \Delta\mathbf{B}(\mu) = \begin{bmatrix} \Delta B(\mu) \\ 0_{n_u} \end{bmatrix} \\ \mathbf{A}(\mu) = \begin{bmatrix} A(\mu) & B(\mu) \\ 0_{n_u\times n_x} & I_{n_u} \end{bmatrix}, \mathbf{L} = \begin{bmatrix} 0_{n_x\times n_u} \\ I_{n_u} \end{bmatrix} \\ \mathbf{C} = \begin{bmatrix} C & 0_{n_y\times n_u} \\ 0_{n_u\times n_x} & I_{n_u} \end{bmatrix}, \mathbf{B}(\mu) = \begin{bmatrix} B(\mu) \\ 0_{n_u} \end{bmatrix} \\ K_a = \begin{bmatrix} 0_{n_u\times n_x} & I_{n_u} \end{bmatrix} \\ \phi_a(K_a\tilde{x}(k)) = K_a\tilde{x}(k) - \mathrm{sat}_a(K_a\tilde{x}(k)). \end{cases}$$

Denoting $\eta(k) = \begin{bmatrix} \tilde{x}^T(k) & x_c^T(k) \end{bmatrix}^T$, we have the augmented closed-loop system

$$\begin{cases} \eta(k+1) = \bar{\mathcal{A}}(\mu)\eta(k) + \mathcal{D}(\mu)w(k) - \bar{\mathcal{E}}^{ca}(\mu)\phi_a(\mathbf{K}_a\eta(k)) \\ \qquad -\mathcal{E}^{cr}(\mu)\phi_r(\mathbf{K}_r(\mu)\eta(k)) \\ z(k) = \mathcal{H}\eta(k) \end{cases} \tag{21}$$

where

$$\begin{cases} \bar{\mathcal{A}}(\mu) = \mathcal{A}(\mu) + \Delta\mathcal{A}(\mu), \bar{\mathcal{E}}^{ca}(\mu) = \mathcal{E}^{ca}(\mu) + \Delta\mathcal{E}^{ca}(\mu) \\ \mathcal{A}(\mu) = \begin{bmatrix} \mathbf{A}(\mu) + \mathbf{L}D^c(\mu)\mathbf{C} & \mathbf{L}C^c(\mu) \\ B^c(\mu)\mathbf{C} & A^c(\mu) \end{bmatrix} \\ \Delta\mathcal{A}(\mu) = \begin{bmatrix} \Delta\mathbf{A}(\mu) & 0_{(n_x+n_u)} \\ 0_{(n_x+n_u)} & 0_{(n_x+n_u)} \end{bmatrix} \\ \mathcal{E}^{ca}(\mu) = \begin{bmatrix} \mathbf{B}(\mu) \\ E^{ca}(\mu) \end{bmatrix}, \mathcal{E}^{cr}(\mu) = \begin{bmatrix} \mathbf{L} \\ E^{cr}(\mu) \end{bmatrix} \\ \Delta\mathcal{E}^{ca}(\mu) = \begin{bmatrix} \Delta\mathbf{B}(\mu) \\ 0_{(n_x+n_u)\times n_u} \end{bmatrix}, \mathcal{D}(\mu) = \begin{bmatrix} \mathbf{D}(\mu) \\ 0_{(n_x+n_u)\times n_w} \end{bmatrix} \\ \mathbf{K}_a = \begin{bmatrix} K_a & 0_{n_u\times(n_x+n_u)} \end{bmatrix}, \mathbf{H} = \begin{bmatrix} H & 0_{n_z\times n_u} \end{bmatrix} \\ \mathbf{K}_r(\mu) = \begin{bmatrix} D^c(\mu)\mathbf{C} & C^c(\mu) \end{bmatrix}, \mathcal{H} = \begin{bmatrix} \mathbf{H} & 0_{n_z\times(n_x+n_u)} \end{bmatrix} \\ \phi_r(\mathbf{K}_r(\mu)\eta(k)) = \mathbf{K}_r(\mu)\eta(k) - \mathrm{sat}_r(\mathbf{K}_r(\mu)\eta(k)). \end{cases}$$

From the formula of the closed-loop lateral path tracking dynamics, it can be seen that two nonlinear terms are introduced to tackle the phenomenon of actuator constraints. To facilitate the system stability and performance analysis, we consider the following sector conditions.

First, we define a polyhedral set $\Omega \subset \Omega_a \cap \Omega_r$, where

$$\Omega_a = \left\{\eta(k) \in \mathcal{R}^{2(n_x+n_u)} \middle| |(\mathbf{K}_a - G_a)\eta| \le \rho_a\right\} \tag{22}$$

$$\Omega_r = \left\{\eta(k) \in \mathcal{R}^{2(n_x+n_u)} \middle| |(\mathbf{K}_r(\mu) - G_r)\eta| \le \rho_r\right\} \tag{23}$$

with appropriately dimensioned matrices G_a and G_r for any membership function vector μ.

Lemma 3.1 [42]: With the functions $\varphi_a(\mathbf{K}_a\eta)$ and $\phi_r(\mathbf{K}_r(\mu)\eta)$ defined above, if $\eta(k) \in \Omega$, the following inequalities hold

$$\varphi_a^T(\mathbf{K}_a\eta)T_a\left[\varphi_a(\mathbf{K}_a\eta) - G_a\eta\right] \le 0$$
$$\phi_r^T(\mathbf{K}_r(\mu)\eta)T_r\left[\varphi_r(\mathbf{K}_r(\mu)\eta) - G_r\eta\right] \le 0$$

for any appropriately dimensioned matrices T_a and T_r, which are diagonal and positive definite.

We now present the main results in this paper. Since two deadzone nonlinearities appear in the closed-loop system in (21), the generalized sector conditions in *Lemma 3.1* and set inclusion conditions are adopted to tackle the nonlinear saturation of the system. The result on stability and robust $\mathcal{H}_\infty$ performance analysis is given as follows.

Theorem 3.1: For the system in (6) with parametric uncertainties and actuator constraints, and the proposed T-S fuzzy output feedback controller in (14), if there exist scalars ε_i and matrices $P_i > 0,\ i \in \{1, 2, \ldots, r\},\ T_a,\ T_r,\ G_a$, and G_r satisfying

$$\begin{bmatrix} \Pi_{1i} & * & * \\ \Pi_{2i} & \Pi_{3l} & * \\ \Pi_{4i} & \Pi_{5i} & \Pi_{6i} \end{bmatrix} < 0 \tag{24}$$

$$\begin{bmatrix} P_i & * \\ \mathbf{K}_a - G_a & \rho_a^2 \end{bmatrix} \ge 0 \tag{25}$$

$$\begin{bmatrix} P_i & * \\ \mathbf{K}_{ri} - G_r & \rho_r^2 \end{bmatrix} \ge 0 \tag{26}$$

where

$$\begin{cases} \Pi_{1i} = \begin{bmatrix} -P_i & * & * & * \\ 0 & -I_{n_w} & * & * \\ T_aG_a & 0 & -2T_a & * \\ T_rG_r & 0 & 0 & -2T_r \end{bmatrix} \\ \Pi_{2i} = \begin{bmatrix} \mathcal{A}_i & \mathcal{D}_i & -\mathcal{E}_i^{ca} & -\mathcal{E}_i^{cr} \\ \mathcal{H} & 0 & 0 & 0 \end{bmatrix} \\ \Pi_{4i} = \left[\begin{array}{c} 0_{n_1\times(2n_x+4n_u+n_w)} \\ \hline \mathcal{S}_i \ \ 0_{n_2\times n_w} \ \ -S_2 \ \ 0_{n_2\times n_u} \end{array}\right] \\ \Pi_{3l} = -\mathrm{diag}\{P_l^{-1}, \lambda^2 I\}, \Pi_{6i} = -\mathrm{diag}\{\varepsilon_i^{-1}I, \varepsilon_i I\} \\ \Pi_{5i} = \mathrm{diag}\{\mathcal{R}_i^T, 0_{n_2\times n_z}\} \end{cases}$$

$$\begin{cases} \mathcal{A}_i = \begin{bmatrix} \mathbf{A}_i + \mathbf{L}D_i^c\mathbf{C} & \mathbf{L}C_i^c \\ B_i^c\mathbf{C} & A_i^c \end{bmatrix} \\ \mathcal{E}_i^{ca} = \begin{bmatrix} \mathbf{B}_i \\ E_i^{ca} \end{bmatrix}, \mathcal{E}_i^{cr} = \begin{bmatrix} \mathbf{L} \\ E_i^{cr} \end{bmatrix} \\ \mathcal{D}_i = \begin{bmatrix} \mathbf{D}_i \\ 0_{(n_x+n_u)\times n_w} \end{bmatrix}, \mathbf{A}_i = \begin{bmatrix} A_i & B_i \\ 0_{n_u\times n_x} & I_{n_u} \end{bmatrix} \\ \mathbf{B}_i = \begin{bmatrix} B_i \\ 0_{n_u} \end{bmatrix}, \mathcal{S}_i = \begin{bmatrix} S_{1i} & S_2 & 0_{n_2\times(n_x+n_u)} \end{bmatrix} \\ \mathbf{D}_i = \begin{bmatrix} D_i \\ 0_{n_u\times n_w} \end{bmatrix}, \mathcal{R}_i = \begin{bmatrix} R_i \\ 0_{(n_x+2n_u)\times n_1} \end{bmatrix}, \end{cases}$$

it guarantees that the closed-loop vehicle steering system (21) is asymptotically stable with robust $\mathcal{H}_\infty$ performance λ in the region $\Omega = \{\eta(k)|\eta^T(k)P(\mu)\eta(k) < 1\}$ ($P(\mu) = \sum_{i=1}^r \mu_i P_i$) for the path with different curves, and here Ω is an approximation of the origin basin of attraction.

Proof: We introduce the fuzzy Lyapunov function as follows:

$$V(k) = \eta^T(k)P(\mu)\eta(k), \tag{27}$$

where $P(\mu) = \sum_{i=1}^{r} \mu_i P_i$ and $P_i \in \mathbb{S}^{2(n_x+n_u)} > 0$ are Lyapunov matrices to be determined.

Define $\xi(k) = \left[\, \eta^T(k)\ w^T(k)\ \varphi_a^T(\mathbf{K}_a\eta)\ \varphi_r^T(\mathbf{K}_r(\mu)\eta) \,\right]^T$ and $\triangle V(\mu,k) = V(\mu^+, k+1) - V(\mu,k)$, where $\mu^+ = \mu(\varsigma(k+1))$ denotes the membership function vector at the time instant $(k+1)T_s$. To show the closed-loop system with actuator constraints satisfying the requirements mentioned above, it is sufficient to guarantee that the condition $\mathcal{J}_1(k) = \triangle V(k) + \frac{z^T(k)z(k)}{\gamma^2} - w^T(k)w(k) < 0$ holds. With the sector conditions introduced in *Lemma 3.1*, one concludes $\mathcal{J}_1(k) < 0$ for $\eta(k) \in \Omega_a \bigcap \Omega_r$ when the inequality holds:

$$\begin{aligned}\mathcal{J}_2(k) &= \triangle V(k) + \frac{z^T(k)z(k)}{\lambda^2} - w^T(k)w(k)\\ &\quad -2\phi_r^T(\mathbf{K}_r(\mu)\eta)T_r\left[\varphi_r(\mathbf{K}_r(\mu)\eta) - G_r\eta\right]\\ &\quad -2\varphi_a^T(\mathbf{K}_a\eta)T_a\left[\varphi_a(\mathbf{K}_a\eta) - G_a\eta\right] < 0. \end{aligned} \tag{28}$$

Along the trajectory of (21), the right-hand-side (RHS) of (28) is reformulated as:

$$\begin{aligned}\mathcal{J}_2(k) &= \eta^T(k+1)P(\mu^+)\eta(k+1) - \eta^T(k)P(\mu)\eta(k)\\ &\quad + \frac{z^T(k)z(k)}{\lambda^2} - w^T(k)w(k)\\ &\quad - 2\varphi_a^T(\mathbf{K}_a\eta)T_a\left[\varphi_a(\mathbf{K}_a\eta) - G_a\eta\right]\\ &\quad - 2\phi_r^T(\mathbf{K}_r(\mu)\eta)T_r\left[\varphi_r(\mathbf{K}_r(\mu)\eta) - G_r\eta\right]\\ &= \sum_{l=1}^{r}\mu_l^+\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\xi^T(k)\\ &\quad \times\left\{\Pi_{1i} + \bar{\Pi}_{2i}^T\bar{\Pi}_{3l}\bar{\Pi}_{2j}\right\}\xi(k),\end{aligned}$$

where

$$\begin{aligned}\bar{\Pi}_{2i} &= \begin{bmatrix} \bar{\mathcal{A}}_i & \mathcal{D}_i & -\bar{\mathcal{E}}_i^{ca} & -\mathcal{E}_i^{cr}\\ \mathcal{H} & 0 & 0 & 0\end{bmatrix}\\ \bar{\mathcal{A}}_i &= \mathcal{A}_i + \Delta\mathcal{A}_i, \bar{\mathcal{E}}_i^{ca} = \mathcal{E}_i^{ca} + \Delta\mathcal{E}_i^{ca}\\ \Delta\mathcal{A}_i &= \mathcal{R}_iF(k)\mathcal{S}_i, \Delta\mathcal{E}_i^{ca} = \mathcal{R}_iF(k)S_2\\ \bar{\Pi}_{3l} &= \text{diag}\{P_l, \frac{1}{\lambda^2}I\}.\end{aligned}$$

According to *Lemma 2.1* together with Schur complement operation, it is sufficient to guarantee that the matrix $\Pi_{1i} + \bar{\Pi}_{2i}^T\bar{\Pi}_{3l}\bar{\Pi}_{2j} < 0$ by condition (24), which further implies $\mathcal{J}_2(k) < 0$ and thus $\mathcal{J}_1(k)$ with the nonnegativity property of fuzzy membership functions.

From the above stability and performance analysis procedure, two sector conditions are introduced to deal with the nonlinear terms in the closed-loop system. To guarantee that the sector conditions are satisfied, now we have to show the estimate of origin basin attraction Ω belongs to the set $\Omega_a \bigcap \Omega_r$.

If condition (25) holds, one has

$$\sum_{i=1}^{r}\mu_i\begin{bmatrix} P_i & *\\ \mathbf{K}_a - G_a & \rho_a^2\end{bmatrix} \geq 0,$$

which further implies

$$\begin{bmatrix} P(\mu) & *\\ \mathbf{K}_a - G_a & \rho_a^2\end{bmatrix} \geq 0.$$

According to Schur complement, the following inequality can be easily guaranteed

$$P(\mu) - \frac{(\mathbf{K}_a - G_a)^T(\mathbf{K}_a - G_a)}{\rho_a^2} \geq 0.$$

Thus, we conclude that any $\eta(k)$ inside $\Omega = \left\{\eta(k)|\eta^T(k)P(\mu)\eta(k) \leq 1\right\}$ also belongs to Ω_a, i.e., $\Omega \subset \Omega_a$. Following the similar line, we also have $\Omega \subset \Omega_r$ from the condition (26) and then $\Omega \subset \Omega_a \bigcap \Omega_r$. Thus, the proof is completed.

With the closed-loop system stability and $\mathcal{H}_\infty$ performance analysis in *Theorem 3.1*, now we are in the position to present the design result on the parameter computation of the output feedback compensator in (14).

Theorem 3.2: For the steering control system in (6) with actuator constraints and the T-S fuzzy output feedback compensator in (14), if some appropriately dimensioned matrices $X, Y, W, \tilde{P}_i, U_i, U_i, \tilde{A}_i^c, \tilde{B}_i^c, \tilde{C}_i^c, D_i^c, \tilde{E}_i^{ca}, \tilde{E}_i^{cr}, \tilde{G}_{a1}, \tilde{G}_{a2}, \tilde{G}_{r1}$, and $\tilde{G}_{r2}$, $i \in \{1, 2, \cdots r\}$ exist such that the matrix inequalities below hold

$$\begin{bmatrix} \tilde{\Pi}_{1i} & * & *\\ \tilde{\Pi}_{2i} & \tilde{\Pi}_{3l} & *\\ \tilde{\Pi}_{4i} & \tilde{\Pi}_{5i} & \Pi_{6i}\end{bmatrix} < 0 \tag{29}$$

$$\begin{bmatrix} \tilde{P}_i & * & *\\ U_i & V_i & *\\ (K_aX - \tilde{G}_{a1}) & (K_a - \tilde{G}_{a2}) & \rho_a^2\end{bmatrix} \geq 0 \tag{30}$$

$$\begin{bmatrix} \tilde{P}_i & * & *\\ U_i & V_i & *\\ (\tilde{C}_i^c - \tilde{G}_{r1}) & (D^c\mathbf{C} - \tilde{G}_{r2}) & \rho_r^2\end{bmatrix} \geq 0 \tag{31}$$

where

$$\left\{\begin{aligned}\tilde{\Pi}_{1i} &= \begin{bmatrix} -\tilde{P}_i & * & * & * & *\\ -U_i & -V_i & * & * & *\\ 0 & 0 & -I_{n_w} & * & *\\ \tilde{G}_{a1} & \tilde{G}_{a2} & 0_{n_u\times n_w} & -2\tilde{T}_a & *\\ \tilde{G}_{r1} & \tilde{G}_{r2} & 0_{n_u\times n_w} & 0_{n_u\times n_u} & -2\tilde{T}_r\end{bmatrix}\\ \tilde{\Pi}_{2i} &= \left[\begin{matrix} \mathbf{A}_iX + \mathbf{L}\tilde{C}_i^c & \mathbf{A}_i + \mathbf{L}D_i^c\mathbf{C}\\ \tilde{A}_i^c & Y^T\mathbf{A}_i + \tilde{B}_i^c\mathbf{C}\\ \mathbf{H}X & \mathbf{H}\end{matrix}\right.\\ &\qquad\qquad \left.\begin{matrix} \mathbf{D}_i & -\mathbf{B}_i\tilde{T}_a & -\mathbf{L}\tilde{T}_r\\ Y^T\mathbf{D}_i & -\tilde{E}_i^{ca} & -\tilde{E}_i^{cr}\\ 0_{n_z\times n_w} & 0_{n_z\times n_u} & 0_{n_z\times n_u}\end{matrix}\right]\\ \tilde{\Pi}_{3l} &= \begin{bmatrix} \tilde{P}_l - X - X^T & * & *\\ U_l - I - W^T & V_l - Y - Y^T & *\\ 0_{n_z\times(n_x+n_u)} & 0_{n_z\times(n_x+n_u)} & -\lambda^2 I\end{bmatrix}\end{aligned}\right.$$

$$\left\{\begin{aligned}\tilde{\Pi}_{4i} &= \left[\begin{array}{c} 0_{n_1\times(2n_x+4n_u+n_w)}\\ \hline \mathbf{S}_iX\ \ \mathbf{S}_i\ \ 0_{n_2\times n_w}\ \ -S_2\tilde{T}_a\ \ 0_{n_2\times n_u}\end{array}\right]\\ \tilde{\Pi}_{5i} &= \left[\begin{array}{c} \mathbf{R}_i^T\ \ \mathbf{R}_i^TY\ \ 0_{n_1\times n_z}\\ \hline 0_{n_1\times(2n_x+n_u+n_z)}\end{array}\right],\end{aligned}\right.$$

the autonomous vehicle could follow different types of reference path with $w(k) \in [0, \infty)$ and the lateral offset satisfies the condition (19), and moreover, the output feedback controller gains can be computed by

$$\begin{cases} MN^T = W - X^TY \\ B_i^c = N^{-1}(\tilde{B}_i^c - Y^T\mathbf{L}D_i^c) \\ C_i^c = (\tilde{C}_i^c - D_i^c\mathbf{C}X)(M^T)^{-1} \\ A_i^c = N^{-1}(\tilde{A}_i^c - Y^T\mathbf{A}_iX - \tilde{B}_i^c\mathbf{C}X \\ \qquad\qquad -Y^T\mathbf{L}C_i^cM^T)(M^T)^{-1} \\ E_i^{ca} = N^{-1}(\tilde{E}_i^{ca}\tilde{T}_a^{-1} - Y^T\mathbf{B}_i) \\ E_i^{cr} = N^{-1}(\tilde{E}_i^{cr}\tilde{T}_r^{-1} - Y^T\mathbf{L}). \end{cases} \tag{32}$$

Proof: First, we define a nonsingular matrix as follows

$$Q = \begin{bmatrix} Y & ? \\ N^T & ? \end{bmatrix},$$

where $Y, N \in \mathbb{R}^{(n_x+n_v)\times(n_x+n_v)}$, and '?' represents the unimportant elements. Assume the inverse matrix of Q and a transformation one are, respectively, given as

$$Q^{-1} = \begin{bmatrix} X & ? \\ M^T & ? \end{bmatrix}, \Xi = \begin{bmatrix} X & I \\ M^T & 0 \end{bmatrix}.$$

Obviously, we have

$$Q\Xi = \begin{bmatrix} I & Y \\ 0 & N^T \end{bmatrix}, \Xi^TQ\Xi = \begin{bmatrix} X^T & W \\ I & Y \end{bmatrix}$$
$$W = X^TY + MN^T.$$

Multiplying (24) on both sides by $diag\{\Xi, I, T_a^{-1}, T_r^{-1}, Q\Xi, I, I, I\}$ on both and its transpose, we can guarantee (24) by (29) with $Q^TP_i^{-1}Q \geq Q^T + Q - P_i$ and the following matrix variable transformations:

$$\begin{cases} \Xi^TP_i\Xi = \begin{bmatrix} \tilde{P}_i & U_i^T \\ U_i & V_i \end{bmatrix}, G_a\Xi = \begin{bmatrix} \tilde{G}_{a1} & \tilde{G}_{a2} \end{bmatrix} \\ G_r\Xi = \begin{bmatrix} \tilde{G}_{r1} & \tilde{G}_{r2} \end{bmatrix}, \tilde{T}_a = T_a^{-1}, \tilde{T}_r = T_r^{-1} \\ \tilde{A}_i^c = Y^T\mathbf{A}_iX + \tilde{B}_i^c\mathbf{C}X + Y^T\mathbf{L}C_i^cM^T + NA_i^cM^T \\ \tilde{B}_i^c = Y^T\mathbf{L}D_i^c + NB_i^c, \tilde{C}_i^c = D_i^c\mathbf{C}X + C_i^cM^T \\ \tilde{E}_i^{ca} = (Y^T\mathbf{B}_i + NE_i^{ca})T_a^{-1}, \tilde{E}_i^{cr} = (Y^T\mathbf{L} + NE_i^{cr})T_r^{-1}. \end{cases}$$

Similarly, taking a congruence transformation to (25) and (26) with diag$\{Q, I_{n_u}\}$ and considering matrix variable changes defined above, we readily have (30) and (31), which also guarantee these conditions (25) and (26) in *Theorem 3.1*. Thus the proof is completed.

By solving these conditions given in *Theorem 3.2*, an admissible dynamic output feedback controller is derived, which guarantees the vehicle with parametric uncertainties and actuator constraints to track different types of path with a certain $\mathcal{H}_\infty$ performance index. With the controller synthesis procedure, we have the following two remarks.

Remark 3.1: The condition (29) in *theorem 3.2* involves two scalar parameters ε and ε^{-1}. When the parameter ε is prefixed, the condition (29) will be strict linear matrix inequality, and could be solved with Matlab software. In this case, how to determine the value of ε raises to guarantee the feasibility of the condition (29). An easy way is to use the trail-and-error method and the other possible one is to apply some numerical optimization search algorithms, for example, genetic algorithm and the optimization program `fminsearch` in Matlab, which have been proved to be efficient approaches for parameter tuning problems in LMI-based conditions [44], [45].

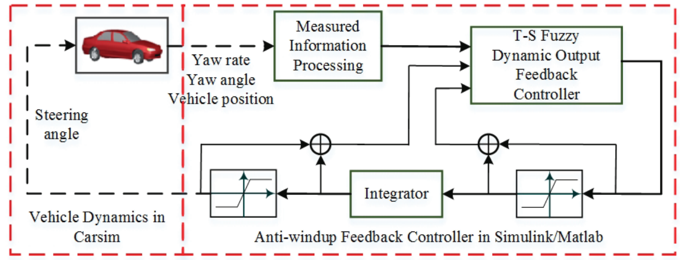


Fig. 2. The connection of the closed-loop path tracking system in Carsim/Matlab joint simulation environment

Remark 3.2: In the present work, the problem of anti-windup output feedback control is studied within fuzzy system theory framework for path tracking. To this end, we resort to the classical parallel distributed compensation (PDC) method for the closed-loop system stability and controller synthesis. However, the obtained results with PDC method can be further relaxed by some other existing analysis methods, such as, nonquadratic stabilization and/or membership-function-dependent analysis strategies [46], [47].

IV. Simulation Validations

In this part, some experimental results are presented to illustrate the effectiveness of the proposed output feedback controller design in this work within the Carsim/Matlab joint simulation environment. To imitate the behavior of nonlinear vehicle dynamics in path tracking, the vehicle simulation software Carsim is introduced. In automotive industry, as a fidelity vehicle simulator, Carsim could quickly and accurately simulate the dynamic behavior of different types of vehicle and thus is widely used for the tests of vehicle handling stability, ride, fuel economy and power, etc. [48]. In Carsim/Matlab joint simulation, the D-Class Sedan is selected as the autonomous vehicle to be controlled, and the proposed controller is realized in Matlab. The structure of the lateral path tracking system is described in Figure 2.

The physical parameters of the D-Class Sedan are given as $m = 1530$kg, $J_z = 4607.0$kg·m^2, $l_f = 1.11$m, $l_r = 1.67$m, and the actuator constraints $\rho_a = \frac{40\pi}{180}(40°)$ and $\rho_r = \frac{5\pi}{180}(5°)$. Considering the uncertain cornering stiffness as mentioned in Section II, we set the two values for front and rear tires to be (92500 ± 7500)N/rad and (83250 ± 6750)N/rad, respectively. In the simulation, we take *Road Course* in Carsim as our reference path, which captures various driving scenarios in the real-world driving. On the other hand as the vehicle velocity changes during cruising, the variation range of the vehicle velocity is pre-defined accordingly. The vehicle velocity is assumed to be bounded by $6 \leq v_x \leq 30$m/s, which is generated according to the road condition, such as, the road curvature. Moreover, the vehicle speed is controlled by a predesigned PI controller embedded in Carsim. Following the fuzzy modeling procedure given in Section 2, the T-S fuzzy lateral vehicle dynamics is given as follows:

Fuzzy Rule R^i: If ς_v is $\mathcal{F}^i$ **Then**

$$\begin{cases} x(k+1) = (A_i + \triangle A_i)x(k) \\ \qquad\qquad +(B_i + \triangle B_i)\delta_f(k) + D_i w(k) \\ y(k) = Cx(k) \\ z(k) = Hx(k), i = 1, 2 \end{cases} \tag{33}$$

where ς_v can be determined by (13), $\mathcal{F}^1 = \frac{1-\varsigma_v}{2}$, $\mathcal{F}^2 = \frac{1+\varsigma_v}{2}$, and the system matrices A_i, $\triangle A_i$, B_i, $\triangle B_i$, D_i can be obtained by replacing v_x with $v_{x\min}$ and $v_{x\max}$, respectively, in (11). Its membership function is depicted in Figure 3. Now we are ready to provide some simulation and comparison results of the path tracking in two different cases: 1) the fixed and varying preview distances; 2) low road friction coefficient condition.

Case I: In some existing results, the problem of path tracking control is considered with fixed preview distance regardless of the variations of vehicle velocity and road conditions. To tackle this problem, we assume that the preview distance varies according to the current velocity as mentioned in *Remark 2.1*. Letting the preview distance $l_s = 0.3v_x$ and $\varepsilon_i = 20$ $(i = 1, 2)$, the Lyapunov matrices and controller gains can be obtained by solving the conditions given in *Theorem 3.2* given as follows

$$P_1 = \begin{bmatrix} 0.4609 & -1.0490 & -0.0556 & -0.1314 & 0.0235 \\ -1.0490 & 7.5929 & -1.7287 & -0.2127 & 0.5563 \\ -0.0556 & -1.7287 & 0.9809 & 0.0823 & -0.2213 \\ -0.1314 & -0.2127 & 0.0823 & 0.6075 & -0.2213 \\ 0.0235 & 0.5563 & -0.2213 & -0.2213 & 0.2645 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 0.5911 & -1.2406 & -0.0356 & -0.1607 & 0.0041 \\ -1.2406 & 8.8135 & -1.8149 & -0.2272 & 0.5380 \\ -0.0356 & -1.8149 & 0.9817 & 0.0926 & -0.3654 \\ -0.1607 & -0.2272 & 0.0926 & 0.5947 & -0.2190 \\ 0.0041 & 0.5380 & -0.3654 & -0.2190 & 0.2689 \end{bmatrix}$$

$$A_1^c = \begin{bmatrix} 0.4379 & 0.0017 & -0.0002 & 0.0001 & 0 \\ 0.0127 & 0.0012 & -0.0001 & 0 & 0 \\ -1.0154 & 0.0013 & 0.0001 & 0 & 0 \\ 16.4693 & 0.0723 & -0.0087 & 0.0025 & 0.0002 \\ -35.3131 & -1.5447 & 0.1088 & -0.0306 & -0.0031 \end{bmatrix}$$

$$A_2^c = \begin{bmatrix} -0.2471 & -0.0008 & -0.0001 & 0 & 0 \\ -0.0040 & 0.0010 & 0 & 0 & 0 \\ -0.5288 & 0.0321 & -0.0015 & 0.0004 & 0.0001 \\ 26.6974 & 0.1830 & 0.0025 & -0.0007 & 0.0004 \\ -93.6585 & -2.4387 & 0.0660 & -0.0169 & -0.0048 \end{bmatrix}$$

$$B_1^c = \begin{bmatrix} 0.0212 & 0.0259 & 0.7866 & -0.0683 \\ -0.0802 & 0.0033 & -0.0723 & -0.0642 \\ -0.1432 & -1.2127 & 2.1147 & -0.0576 \\ -1.2083 & -3.2545 & -25.2069 & -2.6464 \\ 2.81071 & 6.81626 & 2.55843 & 2.2844 \end{bmatrix}$$

$$B_2^c = \begin{bmatrix} 0.1278 & 0.2305 & 1.7729 & -0.0046 \\ -0.1044 & -0.0012 & -0.0468 & -0.0652 \\ -0.2528 & -1.5941 & 1.1663 & -0.7689 \\ -1.7254 & -4.2069 & -40.1817 & -3.0608 \\ 6.66972 & 7.9763 & 151.9269 & 45.3533 \end{bmatrix}$$

$$C_1^c = \begin{bmatrix} 0.9361 & -0.0317 & 0.0018 & -0.0005 & -0.0001 \end{bmatrix}$$
$$C_2^c = \begin{bmatrix} 1.1449 & -0.0320 & 0.0018 & -0.0005 & -0.0001 \end{bmatrix}$$

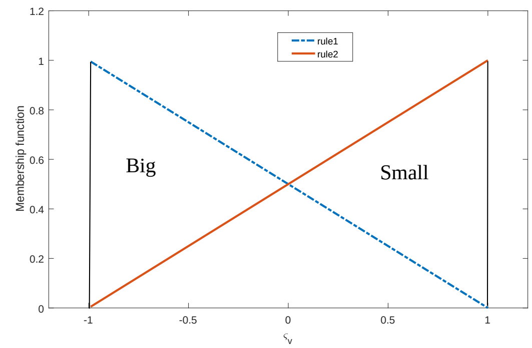


Fig. 3. The membership function for the T-S fuzzy vehicle lateral dynamics

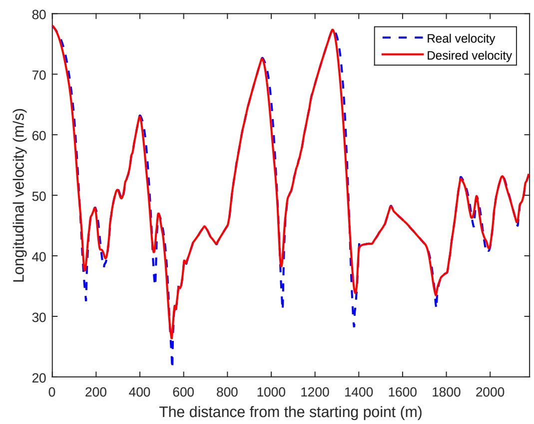


Fig. 4. The longitudinal velocity profile in Case I

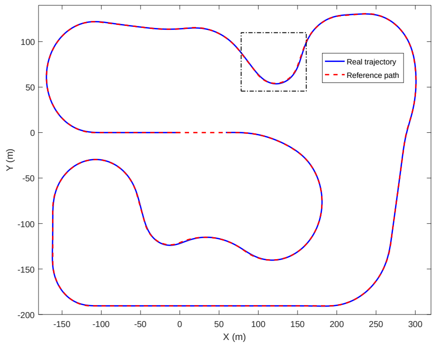
$$D_1^c = \begin{bmatrix} -0.0828 & -0.2943 & -1.5202 & -0.4651 \end{bmatrix}$$
$$D_1^c = \begin{bmatrix} -0.0974 & -0.2832 & -1.7721 & -0.3632 \end{bmatrix}$$

$$E_1^{ca} = \begin{bmatrix} -0.0488 \\ -0.0454 \\ -0.0231 \\ -1.8279 \\ 8.0545 \end{bmatrix}, E_2^{ca} = \begin{bmatrix} -0.0062 \\ -0.0469 \\ -0.1673 \\ -0.9500 \\ 5.2584 \end{bmatrix}$$

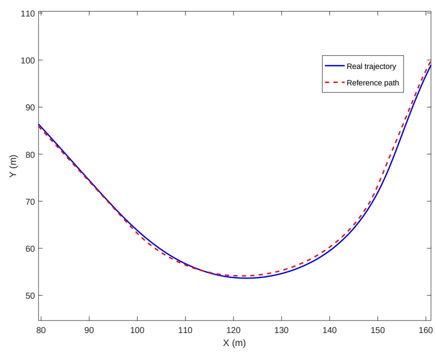
$$E_1^{cr} = \begin{bmatrix} 0.1250 \\ -0.0193 \\ -0.4004 \\ 4.5567 \\ 18.9883 \end{bmatrix}, E_1^{cr} = \begin{bmatrix} -0.0821 \\ -0.0189 \\ -0.7592 \\ 7.0292 \\ 6.7580 \end{bmatrix}.$$

Based on the simulation structure in Figure 2, some experimental results are depicted as follows. In this experiment, the real vehicle velocity ($6 \leq v_x \leq 25$m/s) is shown in Figure 4 (the dashed blue curve), from which we can see that there are usually differences between the desired velocity profile and the real one.

In Figure 5, the reference trajectory of *Road Course* and the real vehicle trajectory are shown, respectively, from which we can find that, with the controller obtained, the vehicle could well track the reference path. To be more specific, the lateral offset is given by the green curve in Figure 7, and we can see that in most sections of the path, the lateral offset is bounded by 0.5m and -0.5m. In Figure 6, we show the measured



(a) The vehicle tracking performance with velocity $6\text{m/s} \leq v_x \leq 25\text{m/s}$



(b) The zoomed-in version of the curves in the block of Fig. 5(a)

Fig. 5. The tracking performance in Case I

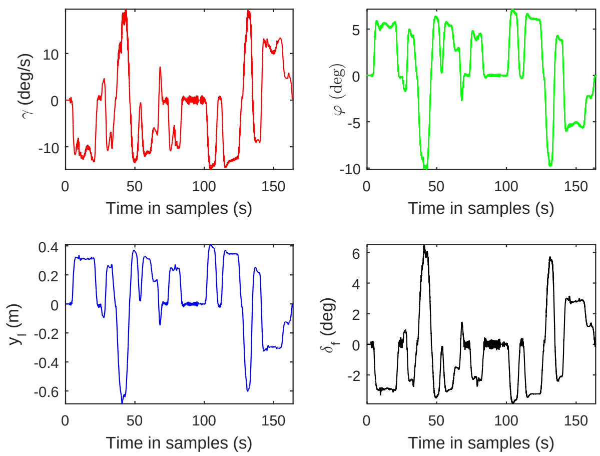


Fig. 6. The measurements and steering angle in Case I

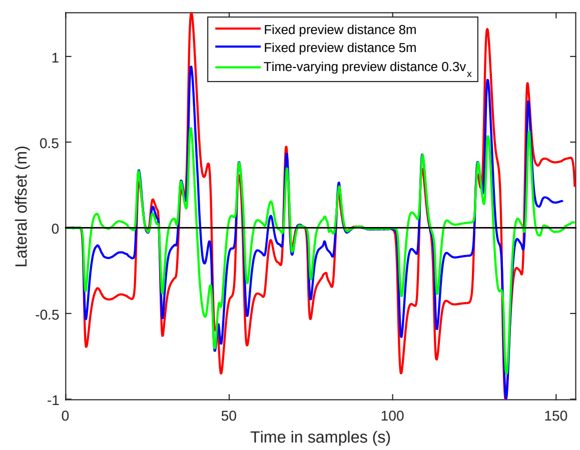


Fig. 7. The lateral offsets under different looking ahead strategies

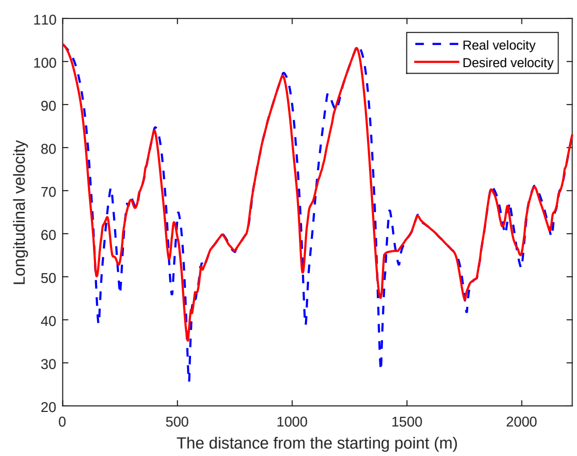


Fig. 8. The longitudinal velocity profile in Case II

and processed outputs of the tracking system and the steering angle input. With these figures shown above, we conclude that the proposed T-S fuzzy output feedback controller could guarantee the vehicle to follow the reference path with time-varying velocities. To further demonstrate the advantages of the varying looking-ahead strategy, we compare the tracking performance in terms of lateral offset with fixed and time-varying preview distances. In Figure 7, the lateral offsets are depicted with fixed preview distances, 8m and 5m, and time-varying preview distance $0.3v_x$, respectively. Obviously, the look-ahead strategy proposed in this paper outperforms over the one with fixed preview distance.

Case II: In this case, we consider the tracking performance under unsatisfactory road condition, i.e., relatively low friction coefficient with a higher vehicle velocity compared with that in *Case I*. The vehicle velocity is depicted in Figure 8, from which we can see that the real speed is about $20\text{km/h} \sim 110\text{km/h}$. In Carsim, the road friction coefficient is set to be 0.8. With the developed fuzzy dynamic output feedback compensator, the tracking performance is shown in Figure 9 with preview distance of $0.5v_x$, from which we can see that the autonomous vehicle could track the given trajecto-

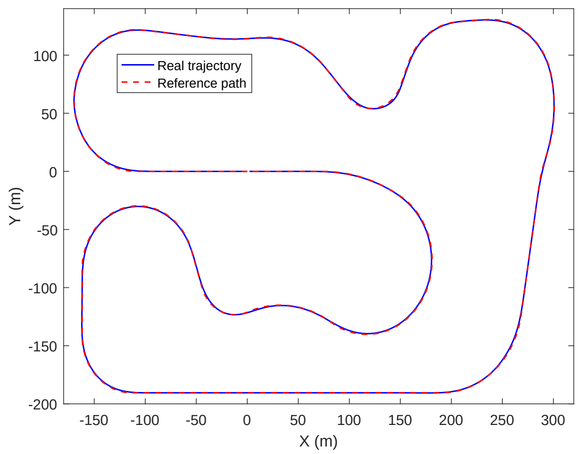
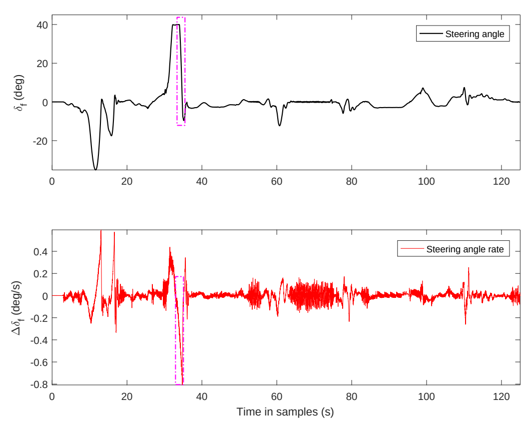
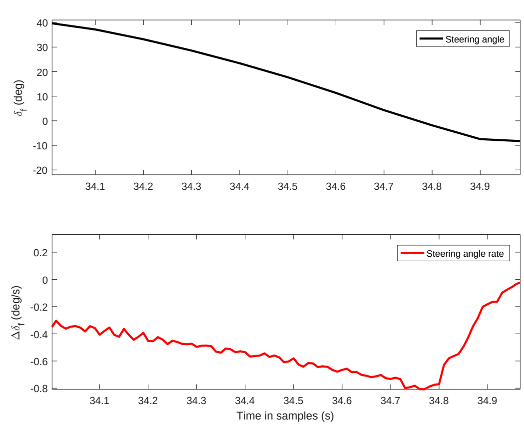


Fig. 9. The tracking performance in Case II



(a) The steering angle and its rate in Case II



(b) The zoomed-in version of the curves in the blocks of Fig. 10(a)

Fig. 10. The control signals in Case II

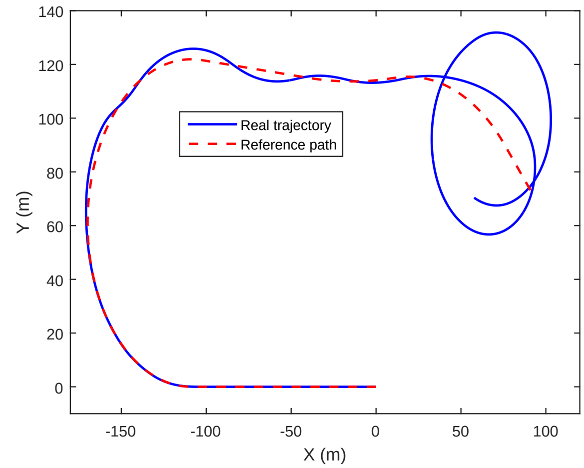


Fig. 11. The tracking performance with Carsim driver model in Case II

ry under low friction coefficient road condition. In Figure 10, the steering angle and its rate are given, respectively, and the actuator amplitude and rate satisfy the aforementioned bounds. To further show the advantage of the proposed controller design method for steering in terms of algorithm robustness, we consider tracking performance of the closed-loop driver model embedded in Carsim, which is set to be without tracking offset under speed varying preview distance $0.5v_x$. In Figure 11, the tracking performance is depicted, and obviously we can find that the vehicle cannot normally follow the path with low friction coefficient, which clearly shows our proposed controller design method is more robust to the vehicle tracking on different road conditions.

V. CONCLUSIONS

In this work, we consider the output feedback steering control for autonomous driving under actuator constraints, i.e., actuator amplitude saturation and rate limit. Considering the nonlinearities and parametric uncertainties, the classical T-S fuzzy modeling method is introduced to represent the nonlinear vehicle lateral model. Within the T-S fuzzy system theory, a method for the T-S fuzzy anti-windup output feedback controller design has been proposed with varying look-ahead distance strategy. With Carsim and Matlab simulation softwares, a series of experimental results are provided to illustrate the effectiveness of the proposed controller design approach. Moreover, some comparison results on tracking performance are also shown under two different cases, i.e., varying or fixed look-ahead distance strategy and low road friction coefficient, which clearly demonstrate the advantages of the proposed control method.

VI. ACKNOWLEDGEMENT

The authors would like to thank the Associate Editor and anonymous reviewers for their constructive comments that have greatly improved the presentation of this paper.

REFERENCES

[1] J. Manigel and W. Leonhard, “Vehicle control by computer vision,” *IEEE Transactions on Industrial Electronics*, vol. 39, no. 3, pp. 181-188, 1992.

[2] D. Åsljung, J. Nilsson, and J. Fredriksson, “Using extreme value theory for vehicle level safety validation and implications for autonomous vehicles,” *IEEE Transactions on Intelligent Vehicles*, vol. 2, no. 4, pp. 288-297, Dec. 2016.

[3] Y. Kang, C. Roh, S.-B. Suh, and B. Song, “A LiDAR-based decision-making method for road boundary detection using multiple Kalman filters,” *IEEE Transactions on Industrial Electronics*, vol. 59, no. 11, pp. 4360-4368, Nov. 2012.

[4] M. Aldibaja, N. Suganuma, and K. Yoneda, “Robust intensity-based localization method for autonomous driving on snow-wet road surface,” *IEEE Transactions on industrial Informatics*, vol. 13, no. 5, pp. 2369-2378, Oct. 2017.

[5] Y. Rasekhipour, A. Khajepour, S.-K. Chen, and B. Litkouhi, “A potential field-based model predictive path-planning controller for autonomous road vehicles,” *IEEE Transactions on Intelligent Transportation Systems*, vol. 18, no. 5, pp. 1255-1267, May 2017.

[6] M. Buehler, K. Iagnemma, and S. Singh, “*The 2005 DARPA grand challenge: the great robot race*,” Springer Science & Business Media, 2007.

[7] L. Alonso, J. Pérez-Oria, B. M. Al-Hadithi, and A. Jiménez, “Self-tuning PID controller for autonomous car tracking in urban traffic,” *The 17th International Conference on System Theory, Control and Computing*, pp.15-20, Sinaia, Romania, 2013.

[8] L. B. Cremean, T. B Foote, J. H. Gillula, G. H. Hines, D. Kogan, K. L. Kriechbaum, J. C. Lamb, J. Leibs, L, Lindzey, C. E. Rasmussen, A. D. Stewart, J. W. Burdick, and R. M. Murray, “Alice: an information-rich autonomous vehicle for high-speed desert navigation,” *Journal of Field Robotics*, vol. 23, no. 9, pp. 777-810, 2006.

[9] R. Marino, S. Scalzi, and M. Netto, “Nested PID steering control for lane keeping in autonomous vehicles,” *Control Engineering Practice*, vol. 19, no. 12, pp. 1459-1467, 2011.

[10] J. H. Ruan, Y. M. Fu, and Y. B. Li, “Intelligent vehicle lateral controller design based on genetic algorithm and T-S fuzzy-neural network,” *Journal of Systems Engineering and Electronics*, vol. 16, no. 2, pp. 382-387, 2005.

[11] T. Hessburg and M. Tomizuka, “Fuzzy logic control for lateral vehicle guidance,” *IEEE Control Systems*, vol. 14, no. 4, pp. 55-63, 1994.

[12] J. Pérez, V. Milanés, and E. Onieva, “Cascade architecture for lateral control in autonomous vehicles,” *IEEE Transactions on Intelligent Transportation Systems*, vo. 12, no. 1, pp. 73-82, 2011.

[13] X. Wang, M. Fu, H. Ma, and Y. Yang, “Lateral control of autonomous vehicles based on fuzzy logic,” *Control Engineering Practice*, vol. 34, pp. 1-17, 2015.

[14] J. Yang and N. N. Zheng, “An expert fuzzy controller for vehicle lateral control,” *The 33th Annual Conference of the IEEE Industrial Electronics Society*, pp. 880-885, Taipei, Taiwan, 2007.

[15] A.-T. Nguyen, C. Sentouh, and J.-C. Popieul, “Fuzzy steering control of autonomous vehicles under actuator saturation: design and experiments,” *Journal of Franklin Institute*, doi:org/10.1016/j.jfranklin.2017.11.027, 2017.

[16] F. Borrelli, P. Falcone, T. Keviczky, J. Asgari, and D. Horvat, “MPC-based approach to active steering for autonomous vehicle systems,” *International Journal of Vehicle Autonomous Systems*, vol. 3, pp. 265-291, 2005.

[17] P. Falcone, F. Borrelli, J. Asgari, H. E. Tseng, and D. Hrovat, “Predictive active steering control for autonomous vehicle systems,” *IEEE Transactions on Control Systems Technology*, vol. 15, no. 3, pp. 566-580, 2007.

[18] G. V. Raffo, G. K. Gomes, J. E. Normey-Rico, C. R. Kelber, and L. B. Becker, “A predictive controller for autonomous vehicle path tracking,” *IEEE Transactions on Intelligent Transportation Systems*, vol. 10, no. 1, pp. 92-102, 2009.

[19] B. Kim, D. Necsulescu, and J. Sasiadek, “Model predictive control of an autonomous vehicle,” *2001 IEEE/ASME International Conference on Advanced Intelligent Mechatronics Proceedings*, pp. 1279-1284, Como, Italy, 2001.

[20] T. Raharijaono, G. DUC, and S. Mammar, “$\mathscr{H}_\infty$ controller synthesis and analysis with application to lateral driving assistance,” *IFAC Symposium on Advance in Automotive Control*, pp. 637-642, Salerno, Italy, 2004.

[21] N. M. Enache, M. Netto, S. Mammar, and B. Lusetti, “Driver steering assistance for lane departure avoidance,” *Control Engineering Practice*, vol. 17, no. 6, pp. 642-651, 2009.

[22] V. Cerone, M. Milanese, D. Regruto, “Combined automatic lane-keeping and driver's steering through a 2-DOF control strategy,” *IEEE Transactions on Control Systems Technology*, vol. 17, no. 1, pp. 135-142, 2009.

[23] M. Lghani, K. Damien, and D.-N Brigitte, “ Discrete robust switched $\mathscr{H}_\infty$ tracking state feedback controller for lateral vehicle control,” *Control Engineering Practice*, vol. 42, pp. 50-59, 2015.

[24] J. Ackermann, “Robust control for automated steering,” *The 29th Conference on Decision and Control*, pp. Honolulu, Hawaii, 1990.

[25] J. Ackermann, J. Guldner, W. Sienel, R. Steinhauser, and V. I. Utkin, “Linear and nonlinear controller design for robust automatic steering,” *IEEE Transactions on Control Systems Technology*, vol. 3, no. 1, pp. 132-143, 1995.

[26] A. P. Aguiar and J. P. Hespanha, “Trajectory-tracking and path following of underactuated autonomous vehicles with parametric modelling uncertainty,” *IEEE Transactions on Automatic Control*, vol. 52, no. 8, pp. 1362-1379, 2007.

[27] R. E. Fenton, G. C. Melocik, and K. W. Olson, “On the steering of automated vehicle: theory and experiment,” *IEEE Transactions on Automatic Control*, vol. 21, no. 3, pp. 306-315, 1976.

[28] J. A. Meda-Campaña, A. Grande-Meza, J. Rubio, R. Tapia-Herrera, T. Hernández-Cortés, A. V. Curtidor-López, L. A. Páramo-Carranza, I. O. Cázares-Ramírez, “Design of stabilizers and observers for a class of multivariable T-S fuzzy models on the basis of new interpolation functions,” *IEEE Transactions on Fuzzy Systems*, vol. 26, no. 5, pp. 2649-2662, 2018.

[29] A. Grande-Meza, T. Hernández-Cortés, A. V. Curtidor-López, L. A. Páramo-Carranza, R. Tapia, I. O. Cázares-Ramírez, J. A. Meda-Campaña, “Analysis of fuzzy observability property for a class of T-S fuzzy models,” *IEEE Latin America Transactions*, vol. 15, no. 4, pp. 595-602, 2017.

[30] J. A. Meda-Campaña, “On the estimation and control of nonlinear systems with parametric uncertainties and noisy outputs,” *IEEE Access*, vol. 6, pp. 31968-31973, 2018.

[31] L. A. Páramo-Carranza, J. A. Meda-Campaña, J. Rubio, R. Tapia-Herrera, A. V. Curtidor-López, “Discrete-time Kalman filter for Takagi-Sugeno fuzzy models,” *Evolving Systems*, vol. 8, no.3, pp. 211-219, 2017.

[32] M. Burckhardt, *Frhrwerktechnik: Radschlupfregelsysteme*, Würzburg, Germany: Vogel-Verlag, 1993

[33] H. B. Pacejka and E. Bakker, “The Magic formula tyre model,” *Vehicle System Dynamics*, vol. 21, no. S1, pp. 1-18, 1992.

[34] J. Dugoff, P. Fanches, and L. Segel, “An analysis of tire properties and their influence on vehicle dynamic performace,” SAE paper (700377), 1970.

[35] J. Kim, “Identification of lateral tyre force dynamics using an extended Kalman filter from experimental road test data,” *Control Engineering Practice*, vol. 17, no. 3, pp. 357-367, 2009.

[36] G. Baffet, A. Charara, and G. Dherbomez, “An observer of tire-force and friction for active security vehicle systems,” *IEEE/ASME Transactions on Mechatronics*, vol. 12, no. 6, pp. 651-661, 2007.

[37] K. Tanaka and H. O. Wang, *Fuzzy control systems design and analysis: a linear matrix inequality approach*, Wiley, 2004.

[38] W. Zong, C. Zhang, Z. Wang, J. Zhu, and Q. Chen, “Architecture design and implementation of an autonomous vehicle,” *IEEE Access*, vol. 6, pp. 21956-21970, 2018.

[39] J. M, Gomes da Silva, D. Limon, T. Alamo, and E. F. Camacho, “Dynamic output feedback for discrete-time systems under amplitude and rate actuator constraints,” *IEEE Transactions on Automatic Control*, vol. 53, no. 10, pp. 2367-2372, 2008.

[40] J. Li, H. O. Wang, D. Niemann, and K. Tanaka, “Dynamic parallel distributed compensation for Takagi-Sugeno fuzzy systems: an LMI approach,” *Information Sciences*, vol. 123, pp. 201-221, 2000.

[41] L. Xie, “Output feedback $\mathcal{H}_\infty$ control of systems with parameters uncertainty,” *International Journal of Control*, vol. 63. no. 4, pp. 741-750, 1996.

[42] T. Zhang, G. Feng, H. Liu, and J. Lu, “Piecewise fuzzy anti-windup dynamic output feedback control of nonlinear processes with amplitude and rate actuator saturations,” vol. 17. no. 2, pp. 253-264, Apr. 2009.

[43] U. Kiencke and L. Nielsen, *Automotive Control Systems for Engine, Driveline, and Vehicle*, 2nd edition, Germany: Springer, 2005.

[44] J. Qiu, G. Feng, and H. Gao, “Observer-based piecewise affine output feedback controller synthesis of continuous-time T-S fuzzy affine

dynamic systems using quantized measurements," *IEEE Transactions on Fuzzy Systems*, vol. 20, no. 6, pp. 1046-1062, Dec. 2012.

[45] U. Shaked, "Improved LMI representations for the analysis and the design of continuous-time systems with polytopic type uncertainty," *IEEE Transactions on Automatic Control*, vol. 46, no. 4, pp. 652-656, Apr. 2001.

[46] T. M. Guerra and L. Vermeiren, "LMI-based relaxed nonquadratic stabilization conditions for nonlinear systems in the Takagi-Sugeno's form," *Automatica*, vol. 40, no. 5, pp. 823-829, 2004.

[47] H. K. Lam and H. Li, "Output-feedback tracking control for polynomial fuzzy-model-based control systems," *IEEE Transactions on Industrial Electronics*, vol. 60, no. 12, pp. 5830-5840, Dec. 2013.

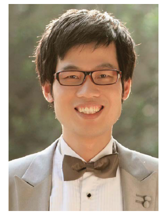
[48] J. Snider, "Automatic steering methods for autonomous automobile path tracking," Robotics Institute, Carnegie Mellon University, Pittsburgh, PA, USA, Tech. Rep. CMU-RI-09-08, 2009.

Changzhu Zhang (M'11) received the B.S. degree in Automation from Qufu Normal University, Rizhao, China, in 2007, the M.S. degree in Control Science and Engineering from Harbin Institute of Technology (HIT), Harbin, China, in 2009, and the Ph.D. degree in Mechatronics Engineering from City University of Hong Kong, Kowloon, Hong Kong, in 2012, respectively. From March 2013 to February 2014, he was with the Institute of Advanced Study, Tongji University, Shanghai, China, as an Associate Research Fellow. From December 2017 to now, he is a Research Fellow with the Department of Informatics, King's College London, London, United Kingdom. Since 2014, he has been with the College of Electrical and Information Engineering, Tongji University, Shanghai, China, where he is currently an Associate Professor. His research interests include intelligent control, networked control systems, signal processing, and autonomous driving.

Hak-Keung Lam (M'98-SM'10) received the B.Eng. (Hons.) and Ph.D. degrees from the Department of Electronic and Information Engineering, The Hong Kong Polytechnic University, Hong Kong, in 1995 and 2000, respectively. During the period of 2000 and 2005, he worked with the Department of Electronic and Information Engineering at The Hong Kong Polytechnic University as Post-Doctoral Fellow and Research Fellow respectively. He joined as a Lecturer at Kings College London in 2005 and is currently a Reader.

His current research interests include intelligent control and computational intelligence. He has served as a program committee member, international advisory board member, invited session chair and publication chair for various international conferences and a reviewer for various books, international journals and international conferences. He is an associate editor for IEEE Transactions on Fuzzy Systems, IEEE Transactions on Circuits and Systems II: Express Briefs, IET Control Theory and Applications, International Journal of Fuzzy Systems and Neorocomputing; and guest editor for a number of international journals. He is in the editorial board of a number of international journals and is an IEEE senior member.

He is a coeditor of two edited volumes: Control of Chaotic Nonlinear Circuits (World Scientific, 2009) and Computational Intelligence and Its Applications (World Scientific, 2012), and author/coauthor of three monographs: Stability Analysis of Fuzzy-Model-Based Control Systems (Springer, 2011), Polynomial Fuzzy Model Based Control Systems (Springer, 2016) and Analysis and Synthesis for Interval Type-2 Fuzzy-Model-Based Systems (Springer, 2016).



Jianbin Qiu (M'10-SM'15) received the B.Eng. and Ph.D. degrees in Mechanical and Electrical Engineering from the University of Science and Technology of China, Hefei, China, in 2004 and 2009, respectively. He also received the Ph.D. degree in Mechatronics Engineering from the City University of Hong Kong, Kowloon, Hong Kong, in 2009.

He is currently a Full Professor at the School of Astronautics, Harbin Institute of Technology, Harbin, China. He was an Alexander von Humboldt Research Fellow at the Institute for Automatic Control and Complex Systems, University of Duisburg-Essen, Duisburg, Germany. His current research interests include intelligent and hybrid control systems, signal processing, and robotics. Prof. Qiu is a Senior Member of IEEE and serves as the chairman of the IEEE Industrial Electronics Society Harbin Chapter, China. He is an Associate Editor of IEEE Transactions on Cybernetics.

Peng Qi (S'11-M'16) received the B.Eng. in Automation from Beijing Jiaotong University, Beijing, China, in 2010 and the M.S. degree in electrical engineering from KTH Royal Institute of Technology, Stockholm, Sweden, in 2012. He obtained the Ph.D. degree in Robotics from Kings College London, United Kingdom, in Feb. 2016.

He was a Research Fellow at the National University of Singapore from Sep. 2015 to Aug. 2016, and a visiting scholar (Honored) at Chinese University of Hong Kong from Sep. 2016 to Feb. 2017. He is currently an Assistant Professor in robotics at Tongji University since October 2016.

Dr. Qi has been selected Shanghai Sailing Program and the Chenguang Program, and involved in some research projects from the EUs Seventh framework programme, National Natural Science Foundation of China, etc. He has published more than 30 scientific research papers, which have been cited more than 500 times in total, and the h-index is 11 (Google Scholar). His current research interests include machine learning, intelligent sensing and interaction, and autonomous intelligent system. Dr. Qi also serves as the representative of China-Britain Artificial Intelligence Association (CBAIA) in China.

Qijun Chen received the B.S. degree in Automatic Control from Huazhong University of Science and Technology, Wuhan, China, in 1987, the M.S. degree in Control Engineering from Xi'an Jiaotong University, Xi'an China, in 1990 and the Ph.D. degree in Control Theory and Control Engineering from Tongji University, Shanghai, China, in 1999, respectively. He was a Visiting Professor with the University of California, Berkeley, CA, USA, in 2008. He is currently a Professor in the College of Electronic and Information Engineering, Tongji University, Shanghai, China. His current research interests include network based control systems and robotics.